NC STATE UNIVERSITY

College of Engineering

Department of Mechanical and Aerospace Engineering

MAE-208, Section 002
Engineering Dynamics

Final Project Design Report

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Problem Statement

The goal of the project is to fabricate a machine comprised of materials that costs no more than $25, can fit within a 3x3x3 foot space, and has no more than two settings. The machine must be able to launch a racquetball at distances in 1 ft increments ranging from 3 to 12 feet, as well as quickly vary the distance it can launch the ball into 5 buckets within a 3-minute time span. The machine must be easily operable so that others can operate it given simple instructions and must not include any electrical power sources must not exceed 9 volts total for the entire machine.

Materials

Table 1, Components and Properties of the Linear and Angular Design

Linear Design Components			**Angular Design Components**		
Component	**Description**	**Properties**	**Component**	**Description**	**Properties**
Spring	Stores conservative energy easily converted to kinetic energy of ball	Length: 6 in Spring Constant: 1.5 lb/in Inner Diameter: .75 in	Spring	Stores conservative energy easily converted to kinetic energy of ball	Length: 4 in Spring Constant: 1.5 lb/in Inner Diameter: .75 in
PVC Pipe	Used to guide ball in launching direction	Length: 1.5 ft Inner Diameter: 2.5 in	Arm	Provides torque to launch ball.	Length: 1.5 ft
Wooden Rod	Pushes ball holder to launch ball	Length: 9 in Outer Diameter: .5 in	Back Arm Stop	Limits the backwards mobility of the arm	Length: 2 in
Plastic Ball Holder	Holds ball in pipe to minimize friction between ball and pipe.	Outer Diameter: 2.4 in Hole Diameter: 2.3	Front Arm Stop	Stops the arm and launches the ball at 45 degrees.	Length: 8 in
Base Plate	Holds and stabilizes launcher	Width: 1 ft Length 2 ft Depth 4 in	Plastic Ball Holder	Holds the ball on the arm	Outer Diameter: 2.4 in Hole Diameter: 2.3
Racquetball	Ball to be launched	Weight: .0875 lbs Diameter: 2.25 in	Racquetball	Ball to be launched	Weight: .0875 lbs Diameter: 2.25 in
			Base Plate	Holds and stabilizes launcher	Width: 1 ft Length 2 ft Depth 4 in

Mechanical Descriptions

Linear Design:
The linear projectile launcher assembly consists of a rod-guided spring mechanism chambered within a pipe and mounted onto a base-plate block. This base plate block is hinged to a U-shaped ground-plate with rubber feet and some amount of weight for anchoring via a vertical hinge. The base plate block is also connected to an arm and vertical hinge, further allowing the device to be aimed at a range of angles to maximize variability of range while denoting set adjustments in order to achieve the goal of hitting varying targets.

It was determined that the best design for accuracy and simplicity would be a compression spring. The spring should sit within the pipe without touching the inner walls. On one end of the spring, an impact face/compression holder will be fixed to the spring. A rod will be connected to this face and run through the spring without touching the spring. This rod will exit the back end of the pipe, which is closed by a stop that is attached to the back end of the spring. To generate a compression force in the spring, this rod will be pulled out in a direction opposite of the pipe. To make the rod easier to move by hand, it will have a handle.

In designing the adjustable arm and vertical hinge, it was determined that three primary angle settings would be desirable. The vertical arm will be attached to a hinge adhered to the baseplate of the pipe at some distance from the muzzle. To fix the arm onto the mounting board during launching, three separate slots are necessary. The slots must ensure the stability of the pipe while launching.

The higher angle would be best for maximizing accuracy at closer ranges, as the higher the launch angle, the larger the relative cross section of the bucket's lid to the moving ball. Notable is that depending on the compression of the spring, the higher launch angle should be viable for medium ranges. The medium launch angle will be the standard angle where the main independent launch variable is the spring compression. Meanwhile, the lower launch angle allows for a further range.

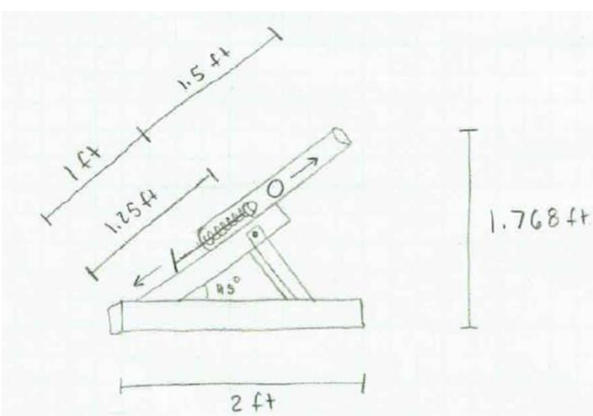


Figure 1, 45° Configuration

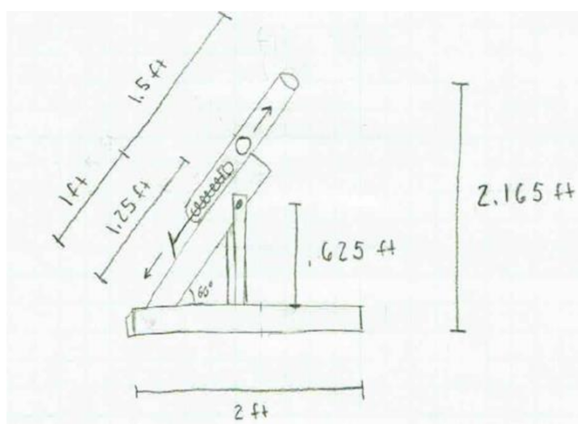


Figure 2, 60° Configuration

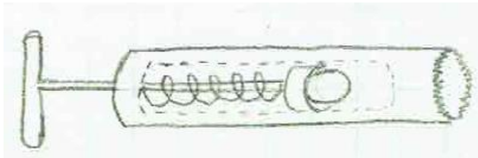


Figure 3, Uncompressed Spring

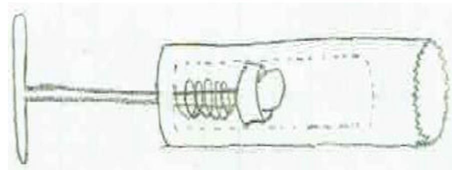


Figure 4, Compressed Spring

Justification for Linear Design Parameters

The spring constant for the linear design was chosen to be 1.5 lb/in with an unstretched length of 6 in. These values were obtained by using the relationship between spring constant and spring displacement: $k = mv^2/s^2$ where k is the spring constant, m is the mass, v is the exit velocity, and s is the spring displacement. Figure 5 below shows a plot of the required spring constant and displacement necessary to launch the ball 12 feet at an angle of 45 degrees since 12 feet is the maximum required distance to launch the ball and 45 degrees is the angle that maximizes the horizontal range of the ball. From this graph, it was determined that a spring constant of 1.5 lb/in and an unstretched length of 6 in would be most appropriate since it would require the spring to be compressed about 3.1 inches to launch the ball 12 feet at a 45 degree angle.

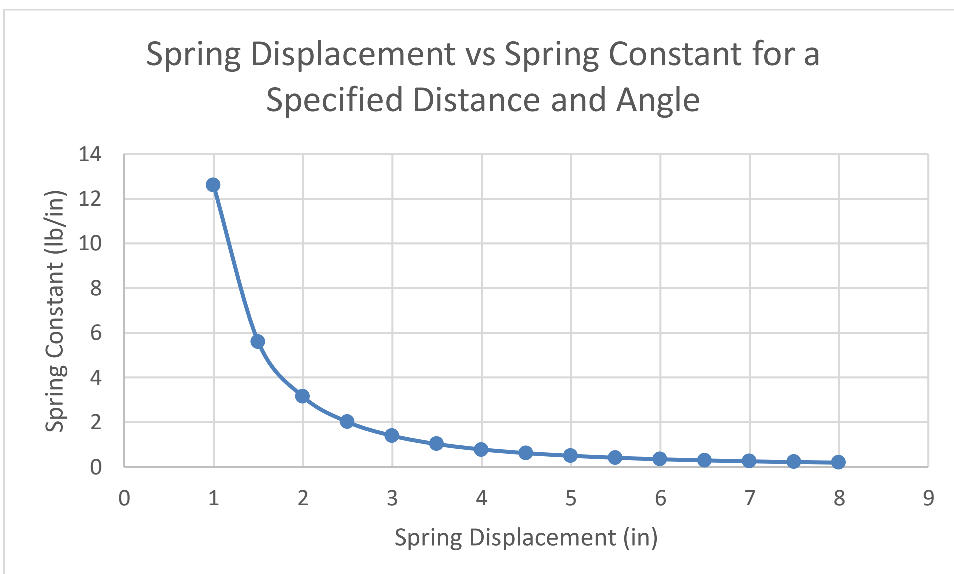


Figure 5, Relationship Between Spring Displacement and Spring Constant Given an Angle of 45° and Desired Distance of 12 ft.

Initially, it was decided that the linear launcher would have 3 fixed launch angles of 30, 45, and 60 degrees. However, the 30 degree angle configuration was later removed from the design as it was concluded that it would not be of any benefit as it would launch a ball the same horizontal distance as the 60 degree angle given the same spring compression. The lower launch angle would also make it harder for the ball to land inside a bucket. Using just the two launch angles of 45 and 60 degrees, the combinations of spring displacement and horizontal distances can be doubled. The required spring displacements to launch the ball at all possible bucket distances at 45 and 60 degrees can be shown in Table -------- in the engineering analysis section below.

The length of the pipe was chosen to be 1.5 ft so that it wouldn't be longer than the 3x3x3 foot space design parameter but long enough to increase accuracy. The inner diameter of the pipe was chosen to be 2.5 in so that the ball could easily fit inside the tube. The PVC material of the pipe was chosen because it would be easy to find at a hardware store and for its smoothness which would reduces friction between the ball in the pipe.

Angular Design:

The angular launcher is a catapult design that comprises of a spring, ball holder, a rotating arm, and an adjustable angle stock so that the ball can be released at multiple angles. The ball rests within the ball holder at the end of the rotating arm which is attached to the spring which generates force when pulled back and released. Thus, the farther the arm is pulled back, the more the spring stretches and the faster the exit velocity of the ball will be. To that end, the distance the projectile is launched is dependent on both angle of release and force generated by the angle of release.

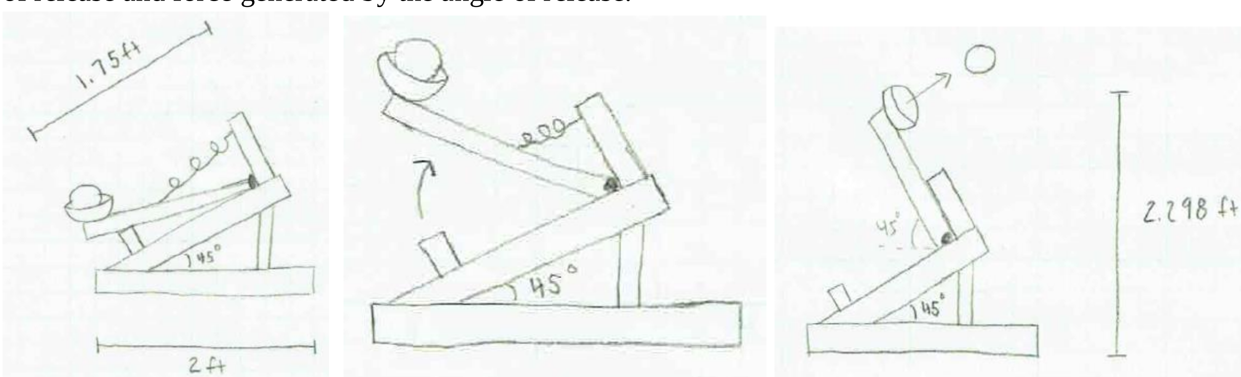


Figure 6, Different Configurations of the Angular Design

Justification of Angular Design Parameters:

The spring for the angular design was chosen to be 6 inches in length with a spring constant of .35 lb/in. The spring constant was chosen to be this low so that the spring could be stretched at higher intervals of stretch which would make it easier to pull back the lever arm to the appropriate position. The lever arm was decided to be 1.5 ft so that it could freely rotate within the 3x3x3 foot space parameter. The catapult was designed at a 45° default angle since this angle would maximize the horizontal distance. Though the catapult is fixed to launch the ball at a 45° angle, the loss in variation is compensated by the low spring constant which allows for more precise measurements of spring stretch.

Engineering Analysis

Linear Design Calculations

Calculating Exit Velocity:

The exit velocity was calculated by setting the potential energy created in compressing the spring to kinetic energy of the ball after release.

Conservation of Energy Formula:

$$mgh_1 + mv_1{}^2 + \frac{1}{2}ks_1{}^2 = mgh_1 + \frac{1}{2}mv_2{}^2 + \frac{1}{2}ks_2{}^2$$

Where m is the mass of the object, v is the velocity, k is the spring constant, and s is the stretch of the spring. The mgh terms are assumed to be zero since the weight of the ball and change in elevation is so small (an elevation change of 1 ft of the ball is only .0875 ft*lb which is relatively small compared to the total energy). Setting the initial kinetic energy and final potential energy to zero, the equation reduces to:

$$\frac{1}{2}ks_1{}^2 = \frac{1}{2}mv_2{}^2$$

Rearranging the terms and solving for v_2:

$$v_2 = s_1\sqrt{\frac{k}{m}}$$

Calculating Horizontal Distance:

The horizontal distance traveled by the ball given an exit velocity and angle was calculated using basic rectilinear motion kinematics.

Let the final velocity of an object undergoing rectilinear acceleration by determined by:

$$v_f = v_o + a_c t$$

Setting the final vertical velocity to zero for when the ball reaches its maximum height, we can solve for the time and multiply this value by two to get the time it takes for the ball to reach the ground:

$$t = \frac{2v_o\sin(\theta)}{a_c}$$

Let the horizontal and vertical displacement be represented by the generic equation below.

$$s = s_o + v_o t + \frac{1}{2}a_c t^2$$

Where s is either horizontal or vertical displacement, s_o is the initial position, v_o is the initial velocity, t is the time in seconds, and a_c is acceleration. Plugging in the value of t calculated above, we can solve for the horizontal displacement of the ball in terms of the exit velocity and the angle at which the ball is launched:

$$x = \frac{2v_o{}^2\sin(\theta)\cos(\theta)}{a_c}$$

The required exit velocity to launch the ball a distance of 12 feet at an angle of 45 degrees is therefore 19.649 ft/s. Plugging this value into the equation for kinetic energy and using .00272 slugs for the mass of the ball, the required potential energy needed is .5249 ft*lbs.

Angular Design Calculations

Conservation of Energy (Curvilinear Motion)

The conservation of energy for the angular design is similar to the linear design but with the addition of .5*I*w^2:

$$mgh_1 + \frac{1}{2}ks_1{}^2 + \frac{1}{2}mv_1{}^2 + \frac{1}{2}I\omega_1{}^2 = mgh_2 + \frac{1}{2}ks_2{}^2 + \frac{1}{2}mv_2{}^2\ \frac{1}{2}I\omega_2{}^2$$

Where I is the moment of inertia and ω is the angular velocity. The moment of inertia for a rod with a point mass "m" and a distance "r" from the axis of rotation is $I = mr^2$ and the velocity is related to angular velocity by $v = \omega r$. The energy related to the weight of the ball is assumed to be small enough to be assumed zero. When the arm is pulled back, the only energy is potential energy at the initial state. After the arm rotes and releases the ball, the only energy is the kinetic energy of the ball so the equation reduces to:

$$\frac{1}{2}ks_1{}^2 = \frac{1}{2}mv_2{}^2$$

Noting that this equation is the same as for the linear design, the exit velocity and horizontal range of the ball can be calculated in the same manor.

Table 2, Required Spring Stretch vs Desired Horizontal Distance of the Ball

	Required Spring Stretch (in)		
Horizontal Distance (ft)	Linear (45° angle)	Linear (60° Angle)	Angular (45° Angle)
3	1.45	1.56	3.00
4	1.67	1.8	3.46
5	1.87	2.01	3.87
6	2.05	2.2	4.24
7	2.21	2.38	4.58
8	2.37	2.54	4.90
9	2.51	2.7	5.20
10	2.65	2.84	5.48
11	2.77	2.98	5.74
12	2.9	3.11	6.00

Description: Table 2 shows the required spring stretch to launch the ball the desired distances for both the linear design configurations and the angular design.

Pros and Cons of Linear vs Angular Design

Both the linear and angular launchers should be able to launch a racquetball varying distances and perform within design requirements. Both designs use a spring to implement a conservative force in order to propel the ball, meaning that they both require the same amount of energy if the ball is released at the same angle. Even though both designs are able to accomplish the designated task, it is important to consider how the aspects of each design affect the accuracy and overall usefulness of the launcher. The linear launcher is characterized by its ability to launch the ball with a predictable and consistent angle and velocity, as well as its ability to launch more balls in a shorter time span. However, the linear launcher also assumes that the friction between the ball holder and the side of the PVC pipe is negligible, which could cause the exit velocity to be much less than predicted. The angular launcher's defining feature is its ability to launch the ball further by minimizing friction that would be present in a launcher barrel (or pipe). However, the angular launcher has a lower accuracy as it is more difficult to calibrate as well as a lower precision as the ball may release at different moments along the rotation of the arm.

Conclusion

Even though the angular launcher does have its benefits, it appears to have enough downsides that makes it significantly worse at accurately and precisely launching the ball to given intervals, while the linear launcher is able to more accurately and precisely hit all the given targets and is much easier to load faster. Given these considerations we can conclude that the linear launcher is better suited to the specified design requirements.

Bibliography

Citations:

"USAR Official Rules & Regulations." *Team USA*, United States Olympic Committee, 2018, www.teamusa.org/USA-Racquetball/rules/2-Courts-and-Equipment.

"Power Transmission." *McMaster-Carr*, McMaster-Carr Supply Company, 2018, www.mcmaster.com/#.

1. MAE 208 Projectile Launcher Project Matlab Code

1.1. Contents

1.2. Linear Projectile Motion (Initial Conditions)

```
k = 1.5; % spring constant (lb/in)
s = 2.5; % spring "stretch" (in)
w = .0875; % weight of object (lb)
g = 32.174; % acceleration due to gravity (ft/s^2)
m = w/g; % mass of object (slugs)
angle = 45; % angle of trajectory (degrees)
```

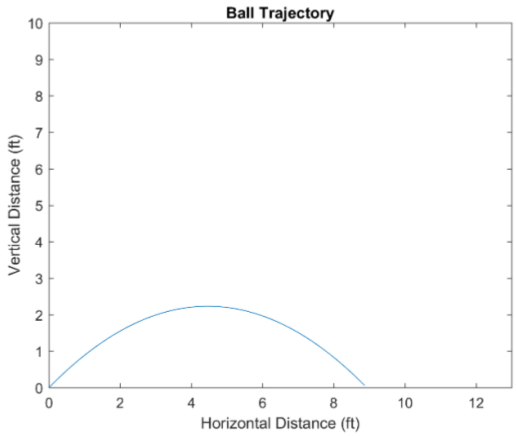
1.3. Linear Projectile Motion Plotter (plots the trajectory of the ball)

```
v1 = ((s/12)*sqrt((k*12)/m)); % velocity of object after spring release (ft/s)
tt = (2*v1*sind(angle))/g; % total time (s)
delta_x = (2*v1^2*sind(angle)*cosd(angle))/g; % horizontal displacement (ft)

t = 0:.01:tt;
x = v1.*cosd(angle).*t;
y = v1.*sind(angle).*t+.5.*-g.*t.^2;
plot(x,y);
axis([0 13 0 10]);
title('Ball Trajectory');
xlabel('Horizontal Distance (ft)');
ylabel('Vertical Distance (ft)');

fprintf('Horizontal distance traveled: %.3f ft\n', delta_x);
```

```
Horizontal distance traveled: 8.929 f
```



1.4. Angle (calculates the required angle given other initial conditions)

```
x1 = input('Desired Distance (ft) = ');
angle1 = acosd(x1/(v1*tt));
```

1.5. Spring Stretch (calculates the required spring stretch given other initial conditions)

```
x1 = input('Desired Distance (ft) = ');
v = sqrt((x1*g)/(2*sind(angle)*cosd(angle)));
s = v/(sqrt((k/12)/m));

fprintf('Required Spring Stretch: %.3f in\n', s);
```

1.6. Table of Required Spring Stretches for Each Possible Bucket Distance

```
for i = 3:12
    x = i;
    v = sqrt((x*g)/(2*sind(angle)*cosd(angle)));
    s = v/(sqrt((k/12)/m));
    table(i-2,1) = i;
    table(i-2,2) = s;
end

display(table);
```